Binding-Blocking Automata

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Binding-Blocking Automata

- Consists of
  - finite control
  - finite tape
  - tape head
  - finite tape symbols
  - transition function
  - partial order relation
  - blocking and unblocking functions

BBA - Formal Definition

- $\mathcal{P} = (Q, V, E, \delta, q_0, R, \beta_b, \beta_{ub}, Q_{accept}, Q_{reject})$,
- $Q = Q_{block} \cup Q_{unblock} \cup Q_{general}$,
- $q_0 \in Q$ (start state), V is a finite set of symbols, E is the finite subset of V^*,
- δ is the transition function from $Q \times E \longrightarrow 2^Q$,
- $R \subseteq E \times E$ is the partial order relation (called as affinity relation) on E,
- β_b is the blocking function from $Q_{block} \longrightarrow 2^V$,
- β_{ub} is the unblocking function from $Q_{unblock} \longrightarrow 2^V$,
- $Q_{accept} \cup Q_{reject} \subseteq Q_{general}$ where Q_{accept} is the set of accepting states and Q_{reject} is the set of rejecting states.

BBA Definition (Contd..)

- The symbols read by the head are called *marked* symbols.
- The symbols blocked are called as *blocked* symbols.
- The head can read a sequence of symbols from its present position.
- Only those symbols which are not marked and not blocked can be read by the head.

Initial Configuration

$$\begin{array}{ccccc} q_0 & a_1 & a_2 & \cdots & a_n \\ \uparrow & - & - & \cdots & - \end{array}$$

Instantaneous Description

$$\begin{array}{ccccccccc} a_1 & a_2 & \cdots & a_{i-1} & q & a_i & a_{i+1} & \cdots & a_n \\ X & X & \cdots & X & \uparrow & Y & Y & \cdots & Y \end{array}$$

Two kinds of transitions

1. l-transition - $X \in \{\#, \$\}$ and $Y \in \{-, \#, \$\}$
2. ll-transition - $X \in \{-, \#, \$\}$ and $Y \in \{-, \#, \$\}$

$q \in Q_{general}$

$$\begin{array}{ccccccccccccc} a_1 & a_2 & \cdots & a_{i-1} & q & a_i & a_{i+1} & \cdots & a_j & a_{j+1} & \cdots & a_n \\ X & X & \cdots & X & \uparrow & - & - & \cdots & - & Y & \cdots & Y \end{array}$$

$$\vdash_l$$

$$\begin{array}{ccccccccccccc} a_1 & a_2 & \cdots & a_{i-1} & a_i & a_{i+1} & \cdots & a_j & p & a_{j+1} & \cdots & a_n \\ X & X & \cdots & X & \# & \# & \cdots & \# & \uparrow & Y & \cdots & Y \end{array}$$

if $\delta(q,x)$ contains p where $x = a_i a_{i+1} \cdots a_j \in V^*$

$q \in Q_{block}$

$$\begin{array}{c}
\cdots \underbrace{\begin{array}{ccc} A & \cdots & A \\ - & \cdots & - \end{array}}_{1} \cdots \underbrace{\begin{array}{ccc} A & \cdots & A \\ - & \cdots & - \end{array}}_{2} \cdots \begin{array}{c} q \\ \uparrow \end{array} \cdots \underbrace{\begin{array}{ccc} A & \cdots & A \\ - & \cdots & - \end{array}}_{r} \\
\vdash_D \\
\cdots \underbrace{\begin{array}{ccc} A & \cdots & A \\ \$ & \cdots & \$ \end{array}}_{1} \cdots \underbrace{\begin{array}{ccc} A & \cdots & A \\ \$ & \cdots & \$ \end{array}}_{2} \cdots \begin{array}{c} p \\ \uparrow \end{array} \cdots \underbrace{\begin{array}{ccc} A & \cdots & A \\ \$ & \cdots & \$ \end{array}}_{r}
\end{array}$$

where $\beta_b(q) \ = \ A$

$q \in Q_{unblock}$

$$\cdots \; A \cdots A \; \cdots \; A \cdots A \; \cdots \; q \; \cdots \; A \cdots A$$

$$\cdots \; \underbrace{\$ \cdots \$}_{1} \; \cdots \; \underbrace{\$ \cdots \$}_{2} \; \cdots \; \uparrow \; \cdots \; \underbrace{\$ \cdots \$}_{r}$$

$$\vdash_l$$

$$p \; \cdots \; A \cdots A \; \cdots \; A \cdots A \; \cdots \; \cdots \; A \cdots A$$

$$\uparrow \; \cdots \; \underbrace{- \cdots -}_{1} \; \cdots \; \underbrace{- \cdots -}_{2} \; \cdots \; \cdots \; \underbrace{- \cdots -}_{r}$$

in case of the leftmost reading and

$$\cdots \; A \cdots A \; \cdots \; A \cdots A \; \cdots \; q \; \cdots \; A \cdots A$$

$$\cdots \; \underbrace{\$ \cdots \$}_{1} \; \cdots \; \underbrace{\$ \cdots \$}_{2} \; \cdots \; \uparrow \; \cdots \; \underbrace{\$ \cdots \$}_{r}$$

$$\vdash_{ll}$$

$$\cdots \; A \cdots A \; \cdots \; A \cdots A \; \cdots \; q \; \cdots \; A \cdots A$$

$$\cdots \; \underbrace{- \cdots -}_{1} \; \cdots \; \underbrace{- \cdots -}_{2} \; \cdots \; \uparrow \; \cdots \; \underbrace{- \cdots -}_{r}$$

in the case of locally leftmost reading.

Language Acceptance and Examples

$$L_D(\mathcal{P}) = \{w \in V^* \mid \begin{array}{cc} q_0 & w \\ \uparrow & - \end{array} \vdash_D^* \begin{array}{cc} w & q_f \\ \# & \uparrow \end{array} q_f \in Q_{accept}\}.$$

Example

$$Q_{general} = \{q_0, q_a, q_b, q_c\},$$

$$Q_{block} = \{q^{block_a}, q^{block_b}, q^{block_c}\},$$

$$Q_{unblock} = \{q^{unblock_a}, q^{unblock_b}, q^{unblock_c}\},$$

$$Q_{accept} = \{q_c\},$$

$$Q_{reject} = \{q_{reject}\},$$

$$\beta_b(q^{block_a}) = \{a\},\ \beta_b(q^{block_b}) = \{b\},\ \beta_b(q^{block_c}) = \{c\},$$

$$\beta_{ub}(q^{unblock_a}) = \{a\},\ \beta_{ub}(q^{unblock_b}) = \{b\},\ \beta_{ub}(q^{unblock_c}) = \{c\},$$

$$R = \{ba > b, ca > c, cb > c\}.$$

Examples (Contd..)

$$\delta(q_0, a) \;=\; \{q^{block_a}\},$$

$$\delta(q^{block_a}, \epsilon) \;=\; \{q_a\},$$

$$\delta(q_a, ba) \;=\; \{q_{reject}\},$$

$$\delta(q_a, b) \;=\; \{q^{unblock_a}\},$$

$$\delta(q^{unblock_a}, \epsilon) \;=\; \{q_{block_b}\}$$

$$\delta(q^{block_b}, \epsilon) \;=\; \{q_b\}$$

$$\delta(q_b, cb) \;=\; \{q_{reject}\},$$

$$\delta(q_b, ca) \;=\; \{q_{reject}\},$$

$$\delta(q_b, c) \;=\; \{q^{unblock_b}\},$$

$$\delta(q^{unblock_b}, \epsilon) \;=\; \{q_{block_c}\}$$

$$\delta(q^{block_c}, \epsilon) \;=\; \{q_c\}$$

$$\delta(q_c, \epsilon) \;=\; \{q_0\},$$

$L \;=\; \{a^n b^n c^n \mid n \geq 1\}$ in ll transition.

Examples (Contd..)

Following is the BBA system $\mathcal{P}$ which accepts

$$L = \{a^n b^n \mid n \geq 1\}$$

$$Q_{general} = \{q_0, q_a, q_b\},$$

$$Q_{block} = \{q^{block_a}, q^{block_b}\},$$

$$Q_{unblock} = \{q^{unblock_a}, q^{unblock_b}\},$$

$$Q_{accept} = \{q_b\},$$

$$\beta_b(q^{block_a}) = a, \ \beta_b(q^{block_b}) = b,$$

$$\beta_{ub}(q^{unblock_a}) = a, \ \beta_{ub}(q^{unblock_b}) = b$$

$$R = \{ba > b, ba > a\}.$$

The transition is defined as follows:

$$\delta(q_0, a) = \{q^{block_a}\},$$

$$\delta(q^{block_a}, \epsilon) = \{q^{unblock_b}\},$$

$$\delta(q^{unblock_b}, \epsilon) = \{q_a\},$$

$$\delta(q_a, ba) = \{q_{reject}\},$$

$$\delta(q_a, b) = \{q^{unblock_a}\},$$

$$\delta(q^{unblock_a}, \epsilon) = \{q_{block_b}\}$$

$$\delta(q^{block_b}, \epsilon) = \{q_0\}.$$

We can easily see that the above system accepts L in both transitions l and ll.

Transition in the above example

The exact transitions done (in the ll transition) by the above system in accepting the word a^3b^3:

$$\begin{array}{ccccccc} q_0 & a & a & a & b & b & b \\ \uparrow & - & - & - & - & - & - \end{array} \longrightarrow \begin{array}{ccccccc} a & q^{block_a} & a & a & b & b & b \\ \# & \uparrow & - & - & - & - & - \end{array} \longrightarrow \begin{array}{ccccccc} a & q^{block_a} & a & a & b & b & b \\ \# & \uparrow & \$ & \$ & - & - & - \end{array}$$

$$\downarrow$$

$$\begin{array}{cccccccc} a & a & a & b & q^{unblock_a} & b & b \\ \# & \$ & \$ & \# & \uparrow & - & - \end{array} \longleftarrow \begin{array}{ccccccc} a & a & a & q_a & b & b & b \\ \# & \$ & \$ & \uparrow & - & - & - \end{array} \longleftarrow \begin{array}{ccccccc} a & a & a & q^{unblock_b} & b & b & b \\ \# & \$ & \$ & \uparrow & - & - & - \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & a & b & q^{unblock_a} & b & b \\ \# & - & - & \# & \uparrow & - & - \end{array} \longrightarrow \begin{array}{ccccccc} a & a & a & b & q^{block_b} & b & b \\ \# & - & - & \# & \uparrow & - & - \end{array} \longrightarrow \begin{array}{ccccccc} a & a & a & b & q^{block_b} & b & b \\ \# & - & - & \# & \uparrow & \$ & \$ \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & a & q^{block_a} & b & b & b \\ \# & \# & \$ & \uparrow & \# & \$ & \$ \end{array} \longleftarrow \begin{array}{ccccccc} a & a & q^{block_a} & a & b & b & b \\ \# & \# & \uparrow & - & \# & \$ & \$ \end{array} \longleftarrow \begin{array}{ccccccc} a & q_0 & a & a & b & b & b \\ \# & \uparrow & - & - & \# & \$ & \$ \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & q^{unblock_b} & a & b & b & b \\ \# & \# & \uparrow & \$ & \# & \$ & \$ \end{array} \longrightarrow \begin{array}{ccccccc} a & a & q^{unblock_b} & a & b & b & b \\ \# & \# & \uparrow & \$ & \# & - & - \end{array} \longrightarrow \begin{array}{ccccccc} a & a & a & b & q_a & b & b \\ \# & \# & \$ & \# & \uparrow & - & - \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & a & b & b & q^{block_b} & b \\ \# & \# & - & \# & \# & \uparrow & - \end{array} \longleftarrow \begin{array}{ccccccc} a & a & a & b & b & q^{unblock_a} & b \\ \# & \# & - & \# & \# & \uparrow & - \end{array} \longleftarrow \begin{array}{ccccccc} a & a & a & b & b & q^{unblock_a} & b \\ \# & \# & \$ & \# & \# & \uparrow & - \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & a & b & b & q^{block_b} & b \\ \# & \# & - & \# & \# & \uparrow & \$ \end{array} \longrightarrow \begin{array}{ccccccc} a & a & q_0 & a & b & b & b \\ \# & \# & \uparrow & - & \# & \# & \$ \end{array} \longrightarrow \begin{array}{ccccccc} a & a & a & b & b & q^{block_a} & b \\ \# & \# & \# & \# & \# & \uparrow & \$ \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & a & b & b & q_a & b \\ \# & \# & \# & \# & \# & \uparrow & - \end{array} \longleftarrow \begin{array}{ccccccc} a & a & a & b & b & q^{unblock_b} & b \\ \# & \# & \# & \# & \# & \uparrow & - \end{array} \longleftarrow \begin{array}{ccccccc} a & a & a & b & b & q^{unblock_b} & b \\ \# & \# & \# & \# & \# & \uparrow & \$ \end{array}$$

$$\downarrow$$

$$\begin{array}{ccccccc} a & a & a & b & b & b & q_b \\ \# & \# & \# & \# & \# & \# & \uparrow \end{array}$$

Definitions

- $A = \{A_1, A_2, \cdots, A_n\}$, $B = \{B_1, B_2, \cdots, B_m\}$, $A_i, B_j \in 2^V$
- A set $S \subseteq V$ is said to be *attainable* from A and B if

  $$S = S_1 * S_2 * \cdots * S_k$$

  where

  $$S_1 \in A, S_i \in A \cup B, i \geq 2 \text{ and}$$

  $$\text{if } S_i \in A \text{ then } * \text{ preceding it is } \cup \text{ or else } * \text{ is } -$$

  The set of all attainable sets is denoted by $\mathcal{A}_V(A, B)$. Note that the evaluation is from left to right.
- A run on BBA is defined as the finite sequence of states $q_0 q_1 q_2 \cdots q_k$ where q_0 is the start state, $q_i \in Q, 1 \leq i \leq k$, $q_k \in Q_{accept} \cup Q_{reject}$ and there exists $a \in V$ such that $q_i \in \delta(q_{i-1}, a)$
- A run is called k-run if k is the the length of the run.
- A run is said to be a block run if $q_1 \in Q_{block}$, and $q_k \in Q_{unblock}$ with $\beta_b(q_1) = \beta_{ub}(q_k) = X$.

Definition

For any BBA system $\mathcal{P}$,

- **q-strings** is the set of all strings $x \in E$ for which $\delta(q, x)$ is non-empty,
- **q-poset** is the poset R induced on the set q-strings.
- Denote the set of all prefixes of q-strings as $Pre(q - strings)$.
- The states q for which the q-poset has only pairs of the form (x, x), $x \in E$ and x is a q-string are called as *unrelated states* and
- The states q for which the q-poset properly includes the pairs (x, x), $x \in E$ and x is q-string are called *related states*.

Notations

- If the affinity relation R is empty then the system is denoted by BBA_{np}
- If the system reads only one symbol at a time then the BBA system is called as a simple BBA system and is denoted by $SBBA$
- The systems by $X_{y,D}$ and the set of languages by $x_{y,D}$ where $X \in \{BBA, SBBA\},\ y \in \{p, np\},\ x \in \{bba, sbba\},\ D \in \{l, ll\}$

Results

BBA_p and BBA_{np} have the same acceptance power.

- Let R_q denote the q-poset for all $q \in Q_{general}$.
- Denote all the components of the poset R_q as $R_q^1, R_q^2, \cdots, R_q^{k_q}$ (if there is only one component, then $k_q = 1$ and $R_q^1 = R_q$).
- Enumerate the elements (strings) in the component R_q^i, $1 \leq i \leq k_q$ as $x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}$ where the enumeration satisfies the condition - for any two strings $x_{i,j}$ and $x_{i,l}$ for $1 \leq j < l \leq k_i$ we have either $(x_{i,j}, x_{i,l}) \notin q$-poset or $x_{i,j} > x_{i,l}$ (Note that there might be more than one such enumeration.). For every related states q, for every component R_q^i we will do the following:

Results (Contd..)

1. Q' includes the set

   $\{q[j, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, y] \mid 1 \leq j \leq k_i, y \in Pre(\{x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}\}), 1 \leq i \leq k_q\} \cup \{q_{dead}\}$

2. For all $p \in Q$ such that $q \in \delta(p, z)$, $z \in E$, $\delta'(p, z)$ includes $q[1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,1,1}]$ where $a_{i,1,1}$ is the first symbol of $x_{i,1}$

3. $\delta'(q[1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,1,1}], a_{i,1,1})$ includes $q[1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,1,1}a_{i,1,2}]$

4. $\delta'(q[1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,1,1} \cdots a_{i,1,m}], a_{i,1,m})$ includes $q[1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,1,1} \cdots a_{i,1,m+1}]$ where $a_{i,1,1} \cdots a_{i,1,m}$ is $PPre(x_{i,1})$

5. If $x_{i,1} = a_{i,1,1} \cdots a_{i,1,m}$ then $\delta'(q[1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,1,1} \cdots a_{i,1,m}], a_{i,1,m})$ includes q' where $\delta(q, x_{i,1})$ includes q'

6. $\delta'(q[j, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,j,1} \cdots a_{i,j,m}], b)$ includes $q[j+1, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,j+1,1}]$ where $b \neq a_{i,j,m}, 1 \leq j < k_i, 1 \leq m \leq \mid x_{i,j} \mid$

7. $\delta'(q[k_i, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,k_i,1} \cdots a_{i,k_i,m}], b)$ contains q_{dead} where $b \neq a_{i,k_i,m}, 1 \leq m \leq \mid x_{i,k_i} \mid$

8. $\delta'(q[j, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,j,1} \cdots a_{i,j,m}], a_{i,j,m})$ includes $q[j, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,j,1} \cdots a_{i,j,m+1}]$ where $m < \mid x_{i,j} \mid, 2 \leq j \leq k_i$

9. $\delta'(q[j, x_{i,1}, x_{i,2}, \cdots, x_{i,k_i}, a_{i,j,1} \cdots a_{i,j,m}], a_{i,j,m})$ contains q' if $x_{i,j} = a_{i,j,1}a_{i,j,2} \cdots a_{i,j,m}$ and $q' \in \delta(q, x_{i,j})$

Result (Contd..)

The acceptance power of BBA is equivalent to that of $SBBA$.

- Without loss of generality we can assume that BBA is a BBA_{np}.
- For every transitions $\delta(q, x)$ with $\mid x \mid = \ k$ where $k \geq 2$ we include states $p_1, p_2, \cdots, p_k$ where $p \in \delta(q, x)$.
- The transitions will be, $p_1 \in \delta(q, \epsilon)$, $p_{i+1} \in \delta(p_i, x_i)$, $1 \leq i \leq k - 1$ and $p \in \delta(p_k, x_k)$ where we have assumed that $x = x_1 x_2 \cdots x_k, x_i \in V, 1 \leq i \leq k$.

Results (Contd..)

k-NFA and k-$SNFA$ are both equivalent.

- Let $M = (k, K, \Sigma, \delta, q_0, \$, F)$ be a k-NFA.
- For each transition with $K \times \{0\}^m \times \{1\}^l \times \{0\}^n \subseteq \delta(q, a_1, a_2, \cdots, a_k)$ where $m + n + l = k, 1 \leq l \leq k, 0 \leq m, n \leq k - 1$ include a serious of l number of transitions in which each of the l heads read a symbol one at a time.
- Let $(q, d_1, d_2, \cdots, d_k) \in \delta(p, a_1, a_2, \cdots, a_k)$ and $i_1, i_2, \cdots, i_j$ with $1 \leq i_s \leq k, 1 \leq s \leq j$ be a strictly monotonically increasing sequence with $d_{i_s} = 1,\ 1 \leq s \leq j$ and assume that there are no more than j one's in the tuple $(q, d_1, d_2, \cdots, d_k)$. Construct a k-$SNFA$, $N = (k, K', \Sigma, \delta', q_0, \$, F)$ as follows:

Results (Contd..)

1. K' includes K,
2. K' includes $\{q_{i_s} \mid 1 \leq s \leq j\}$,
3. $(q_{i_1}, 0, \cdots, 0) \in \delta'(p, a_1, a_2, \cdots, a_k)$ if $(q, d_1, d_2, \cdots, d_k) \in \delta(p, a_1, a_2, \cdots, a_k)$, $d_{i_1} = d_{i_2} = \cdots d_{i_j} = 1$,
4. $(q_{i_s}, 0, \cdots, 0, \underbrace{1}_{i_{s-1}}, 0, \cdots, 0) \in \delta'(q_{i_{s-1}}, a_1, a_2, \cdots, a_k), 1 \leq s \leq j$,
5. $(q, 0, \cdots, 0, \underbrace{1}_{i_j}, 0, \cdots, 0) \in \delta'(q_{i_j}, a_1, a_2, \cdots, a_k), 1 \leq s \leq j$

For each transition in M, we have $j + 1$ transition in N where j denotes the number of heads moving one cell to the right in the that specific transition.

Results (Contd..)

For every language $L \in k\text{-}SNFA$ there is a language $L' \in BBA$ such that L can be written in the form $h^{-1}(L')$ where h is a homomorphism from L to L'.

- Let $M = (k, K, \Sigma, \delta, q_0, \$, F)$ be $k\text{-}SNFA$ accepting the language L.
- Define $L' = \{x^{(k)} \mid x \in L\}$. We have to show that $L' \in BBA(n)$ and there exists a homomorphism h such that $h(L) = L'$.
- Construct a BBA, $\mathcal{P} = (Q, \Sigma^{(k)}, \delta', q_0, \phi, \beta_b, \beta_{ub}, Q_{accept}, \phi)$ as follows:

Results (Contd..)

1. $Q \;=\; K \cup \{q^{block,-i} \mid 1 \leq i \leq k, q \in K\} \cup \{q^{unblock,i} \mid 1 \leq i \leq k, q \in K\}$
2. $q^{block,-i_1} \in \delta'(p,\epsilon)$ if $(q_{i_1}, 0, \cdots, 0) \in \delta(p, a_1, a_2, \cdots, a_k)$
3. $q_{i_1} \in \delta'(q^{block,-i_1}, \epsilon)$
4. $q^{unblock,i_s} \in \delta'(q_{i_{s-1}}, a_{i_{s-1}})$ if $(q_{i_s}, 0, 0, \cdots, \underbrace{1}_{i_{s-1}}, 0, \cdots, 0) \in \delta(q_{i_{s-1}}, a_1, a_2, \cdots, a_k),\; j \geq s \geq 2$
5. $q_{i_s} \in \delta'(q^{unblock,i_s}, \epsilon), 2 \leq s \leq j$
6. $q \in \delta'(q_{i_j}, a_{i_j})$ if $(q_{i_s}, 0, 0, \cdots, \underbrace{1}_{j}, 0, \cdots, 0) \in \delta(q_{i_j}, a_1, a_2, \cdots, a_k),\; j \geq s \geq 2$
7. $\beta_b(q^{block,-i}) \;=\; \{a_1, a_2, \cdots, a_{i-1}, a_{i+1}, \cdots, a_k) \mid a \in \Sigma\}$
8. $\beta_{ub}(q^{unblock,i}) \;=\; \{a_i \mid a \in \Sigma\}$
9. $Q_{accept} \;=\; F$

- We duplicate the symbols in V k times i.e., for each symbol $a \in V$ we have $a_1, a_2, \cdots a_k$.
- When the i^{th} head is active it reads the symbol a_i and all other symbols $b_j, 1 \leq j \neq i \leq k, b \in V$ are blocked.
- When some other gets activate then those symbols corresponding to that head are unblocked and read. It should be easy to see that the system accepts L'.

Results (Contd..)

For every $L \in BBA_l$ there exists BBA_{ll}, $\mathcal{P}$ such that $L(\mathcal{P}) = L$.

- The derivations l and ll are different only when unblocking of symbols takes place.
- In the l derivation the head again starts from the beginning, whereas in the ll derivation the head is unmoved.
- Let $\mathcal{R}$ be the BBA_l.
- We construct BBA_{ll} $\mathcal{P}$ as follows:

Results (Contd..)

1. $q'_0 \;=\; q_0^{\phi}$
2. $Q' \;=\; \{q^X, q^X_{blockall}, q^X_{unblock} \mid q \in Q, X \in \mathcal{B}(\mathcal{P})\}$
3. $p^X \in \delta'(q^X, a)$ if $p \in \delta(q, a), p \in Q_{general}, X \in \mathcal{B}(\mathcal{P}), a \in V \cup \{\epsilon\},$
4. $p^{X \cup \beta_b(p)} \in \delta'(q^X, a)$ if $p \in \delta(q, a), p \in Q_{block}, X \in \mathcal{B}(\mathcal{P}), a \in V \cup \{\epsilon\},$
5. $p^{X - \beta_{ub}(p)}_{blockall} \in \delta'(q^X, a)$ if $p \in \delta(q, a), p \in Q_{unblock}, X \in \mathcal{B}(\mathcal{P}), a \in V \cup \{\epsilon\},$
6. $p^{V-X}_{unblock} \in \delta'(p^X_{blockall}, \epsilon), p \in Q, X \in \mathcal{B}(\mathcal{P}),$
7. $p^{V-Y} \in \delta'(p^Y_{unblock}, \epsilon), p \in Q, X \in \mathcal{B}(\mathcal{P}),$
8. $\beta_b(p^X_{blockall}) \;=\; V,$
9. $\beta_{ub}(p^X_{unblock}) \;=\; X,$

Normal Form of BBA

simple unblocking scheme: $\forall q \in Q_{unblock},\ \beta_{ub}(q) \subseteq \beta_b(p)$ for some $p \in Q_{block}$.

useful blocking scheme: at no time the automaton tries to block an already blocked symbol.

perfect unblocking scheme: $\forall q \in Q_{unblock},\ \beta_{ub}(q) = \beta_b(p)$ for some $p \in Q_{block}$.

A BBA is said to be *well-formed* BBA if it follows both useful blocking and perfect unblocking.

Note: perfect unblocking scheme implies simple unblocking.

Complexity Issues

- **Blocking number** denoted by $n(\mathcal{P})$ is defined as the cardinality of the set

$$\mathcal{A}_V(\beta_b(Q_{block}), \beta_{ub}(Q_{unblock})),$$

Note that the value of $n(\mathcal{P})$ lies between $1 \leq n(\mathcal{P}) \leq \mid 2^V \mid$.

- **Blocking instant** denoted as $B(\mathcal{P})$ is defined as

$$B(\mathcal{P}) \ = \ Max\{Card(A) \mid A \in \mathcal{B}(\mathcal{P})\}$$

- **Blocking quotient** of a set $X \subseteq V$ is defined as the length of the longest run from the blocking of X to the unblocking of X. It is denoted by $BQ_X(\mathcal{P})$.

- **Blocking quotient** of $\mathcal{P}$ is defined as $BQ(\mathcal{P}) \ = \ Max\{BQ_X\}$ where the maximum is taken over all the sets X (where $X \subseteq V$ such that there exists $q \in Q_{block}$ with $\beta_b(q) \ = \ X$). We denote blocking quotient simply as BQ if $\mathcal{P}$ is understood.

- $\mathcal{P}(k, m, n)$ denoted a BBA $\mathcal{P}$ with k the blocking number, m the blocking instant and n blocking quotient.

- For every BBA with infinite blocking quotient there is an equivalent BBA with finite blocking quotient

Results (Contd..)

Given a $BBA, \mathcal{P}$ we can construct an equivalent well-formed $BBA, \mathcal{P}'$ with $L(\mathcal{P}) = L(\mathcal{P}')$.

- BBA with infinite BQ can be converted to a BBA with finite BQ

Let $\mathcal{P} = (Q, V, E, \delta, q_0, R, \beta_b, \beta_{ub}, Q_{accept}, Q_{reject})$. We construct an equivalent BBA_{useb}, $\mathcal{P}_1 = (Q_1, V, E, \delta_1, q_0, R, \beta_b^1, \beta_{ub}^1, Q_{accept}^1, Q_{reject}^1)$ as follows:

1. $Q_1 = \{q^X \mid X \in \mathcal{B}(\mathcal{P})\}$,
2. $q^X \in \delta_1(p^X, a)$ if $q \in \delta(p, a)$ where $p \in Q_{general}$ and $X \in \mathcal{B}(\mathcal{P})$,
3. $q^{X \cup \beta_b(p)} \in \delta_1(p^X, a)$ if $q \in \delta(p, a)$ where $p \in Q_{block}, a \in V \cup \{\epsilon\}$ and $X \in \mathcal{B}(\mathcal{P})$,
4. $q^{X - \beta_{ub}(p)} \in \delta_1(p^X, a)$ if $q \in \delta(p, a)$ where $p \in Q_{unblock}, a \in V \cup \{\epsilon\}$ and $X \in \mathcal{B}(\mathcal{P})$,
5. $\beta_b^1(q^X) = Y - X$ if $\beta_b(q) = Y$,
6. $\beta_{ub}^1(q^X) = Y$ if $\beta_{ub}(q) = Y \cap X$.

Results (Contd..)

The construction of BBA_{sub},

$$\mathcal{P}_2 \;=\; (Q_2, V, E, \delta_2, q_0, R, \beta_b^2, \beta_{ub}^2, Q_{accept}^2, Q_{reject}^2),$$

from the above constructed $BBA_{useb}, \mathcal{P}_1$.

- Let us assume that $\mathcal{P}_1$ has a run $\cdots p \cdots q \cdots r \cdots$ where $p, q \in Q_{block}^1, r \in Q_{unblock}^1, \beta_b^1(p) = X,\ \beta_b^1(q) = Y$ and $\beta_{ub}^1(r) = Z$ with $Z \subseteq X \cup Y$, $Z \cap X \neq \phi$ and $Z \cap Y \neq \phi$.
- Divide the unblocking of Z into two unblocking one as $Z \cap X$ and the other as $Z \cap Y$.
- The system can store the information that which set was blocked first. So the unblocking of Z can be replaced by unblocking of $Z \cap X$ since anyway $\mathcal{P}$ is going to read at least a symbol in X before reading a symbol in Y (otherwise blocking of X is not necessary at that particular place in the run).
- After reading a symbol in X the system can unblock $Z \cap Y$ and the system can proceed as in the original.
- This method can be easily extended to any tuple $(p_1, p_2, \cdots, p_{n-1}, p_n)$ where $p_i \in Q_{block}^1, 1 \leq i \leq n-1, p_n \in Q_{unblock}^1$ with $\beta_b^1(p_i) = X_i, 1 \leq i \leq n-1,\ \beta_{ub}^1(p_n) = Y$, $Y \subseteq \cup_i X_i$ and $Y \cap X_i \neq \phi$.

Results (Contd..)

We show the construction of BBA_{pub},

$$\mathcal{P}' = (Q', V, E, \delta', q_0, R, \beta'_b, \beta'_{ub}, Q'_{accept}, Q'_{reject}),$$

from the above constructed BBA_{sub}.

- Let us assume there is a run $\cdots pr_1r_2\cdots r_kq\cdots$ in $\mathcal{P}_2$ where $r_i \in Q^2, 1 \leq i \leq k, p \in Q^2_{block}$ with $\beta^2_b(p) = X$ and $q \in Q^2_{unblock}$ with $\beta^2_{ub} = Y$ where $Y \subseteq X$.

For each of the above run we do the following

1. $dead, p[freeX - Y], r_i[freeX - Y] \in Q'_{general}, X, Y \subseteq V$,
2. $p[blockX - Y], r_i[blockX - Y] \in Q'_{block}, 1 \leq i \leq k, X, Y \subseteq V$,

   δ' includes all the transitions of δ and the following transitions,
3. $p[freeX - Y], p[blockX - Y] \in \delta'(p, \epsilon)$,
4. $r_i[blockX - Y] \in \delta'(r_i, \epsilon), 1 \leq i \leq k$,
5. $r_i \in \delta'(r_i[blockX - Y], \epsilon)$,
6. $p \in \delta'(p[blockX - Y], \epsilon)$,

7. $r_1[freeX - Y] \in \delta'(p[freeX - Y], a)$ if $a \in V - X$,

8. $r_{i+1}[freeX - Y] \in \delta'(r_i[freeX - Y], a), 1 \leq i \leq k - 1$ if $a \in V - X$,

9. $dead \in \delta'(p[freeX - Y], a)$ if $a \in X - Y$

10. $\beta'_b(p[blockX - Y]) = X - Y$,

Results (Contd..)

- the transition (3) gives a non-deterministic choice of reading symbols with only Y as blocked or blocking $X - Y$ (which gives the same configuration as $\mathcal{P}_2$) and proceed with the transition of $\mathcal{P}_2$.
- For all intermediate transitions in the run between p and q the system includes an ϵ transition to states of the form $s[blockX - Y]$ where $s \in \{p, r_i\}$ from where it can block $X - Y$ and proceed its transition which exactly follows the transition of $\mathcal{P}_2$. These are done by the transitions (4),(5) and (6).
- If suppose the system reads a symbol from $X - Y$ then we have to make sure that the system does not continue reading afterwards (since in $\mathcal{P}_2$ the system can not read any symbol from X. This is taken care of by the transition (9).
- If $X - Y$ is not blocked the system carries on its transition (only when it reads $V - X$ symbols) in the states of the form $r_i[freeX - Y]$.

- The above process can be repeated for all such pairs (p, q) in the run. If this is done for all the runs then it should be easy to see that $L(\mathcal{P}_2) = L(\mathcal{P}')$.
- We note that the above proof of construction of BBA_{sub} fails in the case of ll-transition.

Results (Contd..)

For every $L \in BBA_l$ there is a n-NFA which accepts the same language.

- M will have one head corresponding to every symbol of the alphabet. Initially all heads are at the leftmost symbol.
- When some blocking takes place all the head corresponds to the symbols which are blocked will stop, but other heads will move through until unblocking takes place.
- When unblocking takes place then the control moves to the heads corresponding to the unblocked symbols. These heads can follow the transitions of BBA_l.
- A problem which may arise is that some symbols (which are not blocked at all) might have been read already. So, it has to skip through those symbols. But how many times it has to skip over is the question.

- For that, we can count the number of blocks of symbols (which are not blocked at all) and store it in the state information. So when the unblocking takes place this information can be transferred to the heads corresponding to the unblocked symbols and these heads can skip through the already read symbols and the count can be decremented by one each time it sees a new block (read symbols). The state information can also have the information of which of the symbols are blocked at that instant of time using the the elements of blocking set $\mathcal{B}(\mathcal{P})$.

Summary

- Presented algorithms for solving
  - Hamiltonian path problem
  - Exact 3-set cover problem

  using peptide-antibody interactions
- Showed that peptide computation is universal
- Proposed a model called Binding-Blocking Automata (BBA)
- Defined Normal-form of BBA and studied the acceptance power of BBA

Future Direction

- Defining a system which is universal

  String Binding-Blocking Automata (communicated)

- Representation of binary numbers
- Arithmetical operations using peptides-antibody interactions

  result communicated

Once we accept our limits, we go beyond them.

Albert Einstein

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Thank You!