Complexity Issues in Binding-Blocking Automata

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Peptide Computing

- Uses peptides and antibodies.
- Peptides encrypts the solution space of the problem.
- Antibodies selects the correct solution, by binding to the correct peptides.

Advantages

- Parallel interactions between the peptides and antibodies are possible.
- Highly non-deterministic.
- Makes it possible to solve NP-complete problems in constant bio-steps (efficient).

Binding-Blocking Automata

- Consists of
  - finite control
  - finite tape
  - tape head
  - finite tape symbols
  - transition function
  - partial order relation
  - blocking and unblocking functions

BBA - Formal Definition

- $\mathcal{P} = (Q, V, E, \delta, q_0, R, \beta_b, \beta_{ub}, Q_{accept}, Q_{reject})$,
- $Q = Q_{block} \cup Q_{unblock} \cup Q_{general}$,
- $q_0 \in Q$ (start state), V is a finite set of symbols, E is the finite subset of V^*,
- δ is the transition function from $Q \times E \longrightarrow Q$,
- $R \subseteq E \times E$ is the partial order relation (called as affinity relation) on E,
- β_b is the blocking function from $Q_{block} \longrightarrow 2^V$,
- β_{ub} is the unblocking function from $Q_{unblock} \longrightarrow 2^V$,
- $Q_{accept} \cup Q_{reject} \subseteq Q_{general}$ where Q_{accept} is the set of accepting states and Q_{reject} is the set of rejecting states.

Contd..

- The symbols read by the head are called *marked* symbols.
- The symbols blocked are called as *blocked* symbols.
- The head can read a sequence of symbols from its present position.
- Only those symbols which are not marked and not blocked can be read by the head.

Initial Configuration

$$\begin{array}{ccccc} q_0 & a_1 & a_2 & \cdots & a_n \\ \uparrow & - & - & \cdots & - \end{array}$$

Instantaneous Description

$$\begin{array}{ccccccccc} a_1 & a_2 & \cdots & a_{i-1} & q & a_i & a_{i+1} & \cdots & a_n \\ X & X & \cdots & X & \uparrow & Y & Y & \cdots & Y \end{array}$$

Two kinds of transitions

1. l-transition - $X \in \{\#, \$\}$ and $Y \in \{-, \#, \$\}$
2. ll-transition - $X \in \{-, \#, \$\}$ and $Y \in \{-, \#, \$\}$

Contd..

$q \in Q_{general}$

$$\begin{array}{cccccccccccc} a_1 & a_2 & \cdots & a_{i-1} & q & a_i & a_{i+1} & \cdots & a_j & a_{j+1} & \cdots & a_n \\ X & X & \cdots & X & \uparrow & - & - & \cdots & - & Y & \cdots & Y \end{array}$$

$$\vdash_l$$

$$\begin{array}{cccccccccccc} a_1 & a_2 & \cdots & a_{i-1} & a_i & a_{i+1} & \cdots & a_j & p & a_{j+1} & \cdots & a_n \\ X & X & \cdots & X & \# & \# & \cdots & \# & \uparrow & Y & \cdots & Y \end{array}$$

if $\delta(q, x) = p$ where $x = a_i a_{i+1} \cdots a_j \in V^*$

Contd..

$q \in Q_{block}$

$$\begin{array}{ccccccccccccc} \cdots & A & \cdots & A & \cdots & A & \cdots & A & \cdots & q & \cdots & A & \cdots & A \\ \cdots & \underbrace{- \ \cdots \ -}_{1} & & & \cdots & \underbrace{- \ \cdots \ -}_{2} & & & \cdots & \uparrow & \cdots & \underbrace{- \ \cdots \ -}_{r} \end{array}$$

$$\vdash_D$$

$$\begin{array}{ccccccccccccc} \cdots & A & \cdots & A & \cdots & A & \cdots & A & \cdots & p & \cdots & A & \cdots & A \\ \cdots & \underbrace{\$ \ \cdots \ \$}_{1} & & & \cdots & \underbrace{\$ \ \cdots \ \$}_{2} & & & \cdots & \uparrow & \cdots & \underbrace{\$ \ \cdots \ \$}_{r} \end{array}$$

where $\beta_{ub}(q) = A$

Contd..

$q \in Q_{unblock}$

$$\begin{array}{ccccccccccccccc} \cdots & A & \cdots & A & \cdots & A & \cdots & A & \cdots & q & \cdots & A & \cdots & A \\ \cdots & \underbrace{\$ & \cdots & \$}_{1} & \cdots & \underbrace{\$ & \cdots & \$}_{2} & \cdots & \uparrow & \cdots & \underbrace{\$ & \cdots & \$}_{r} \end{array}$$

$$\vdash_l$$

$$\begin{array}{cccccccccccccc} p & \cdots & A & \cdots & A & \cdots & A & \cdots & A & \cdots & \cdots & A & \cdots & A \\ \uparrow & \cdots & \underbrace{- & \cdots & -}_{1} & \cdots & \underbrace{- & \cdots & -}_{2} & \cdots & \cdots & \underbrace{- & \cdots & -}_{r} \end{array}$$

in case of the leftmost reading and

$$\begin{array}{ccccccccccccccc} \cdots & A & \cdots & A & \cdots & A & \cdots & A & \cdots & q & \cdots & A & \cdots & A \\ \cdots & \underbrace{\$ & \cdots & \$}_{1} & \cdots & \underbrace{\$ & \cdots & \$}_{2} & \cdots & \uparrow & \cdots & \underbrace{\$ & \cdots & \$}_{r} \end{array}$$

$$\vdash_{ll}$$

$$\begin{array}{ccccccccccccccc} \cdots & A & \cdots & A & \cdots & A & \cdots & A & \cdots & q & \cdots & A & \cdots & A \\ \cdots & \underbrace{- & \cdots & -}_{1} & \cdots & \underbrace{- & \cdots & -}_{2} & \cdots & \uparrow & \cdots & \underbrace{- & \cdots & -}_{r} \end{array}$$

in the case of locally leftmost reading.

Contd..

Language Acceptance

$$L_D(\mathcal{P}) = \{w \in V^* \mid \begin{matrix} q_0 & w \\ \uparrow & - \end{matrix} \vdash_D^* \begin{matrix} w & q_f \\ \# & \uparrow \end{matrix} q_f \in Q_{accept}\}.$$

Example

$$Q_{general} = \{q_0, q_a, q_b, q_c\},$$

$$Q_{block} = \{q^{block_a}, q^{block_b}, q^{block_c}\},$$

$$Q_{unblock} = \{q^{unblock_a}, q^{unblock_b}, q^{unblock_c}\},$$

$$Q_{final} = \{q_c\},$$

$$Q_{reject} = \{q_{reject}\},$$

$$\beta_b(q^{block_a}) = \{a\},\ \beta_b(q^{block_b}) = \{b\},\ \beta_b(q^{block_c}) = \{c\},$$

$$\beta_{ub}(q^{unblock_a}) = \{a\},\ \beta_{ub}(q^{unblock_b}) = \{b\},\ \beta_{ub}(q^{unblock_c}) = \{c\},$$

$$R = \{ba > b, ca > c, cb > c\}.$$

Contd..

$$\delta(q_0, a) = \{q^{block_a}\},$$

$$\delta(q^{block_a}, \epsilon) = \{q_a\},$$

$$\delta(q_a, ba) = \{q_{reject}\},$$

$$\delta(q_a, b) = \{q^{unblock_a}\},$$

$$\delta(q^{unblock_a}, \epsilon) = \{q_{block_b}\}$$

$$\delta(q^{block_b}, \epsilon) = \{q_b\}$$

$$\delta(q_b, cb) = \{q_{reject}\},$$

$$\delta(q_b, ca) = \{q_{reject}\},$$

$$\delta(q_b, c) = \{q^{unblock_b}\},$$

$$\delta(q^{unblock_b}, \epsilon) = \{q_{block_c}\}$$

$$\delta(q^{block_c}, \epsilon) = \{q_c\}$$

$$\delta(q_c, \epsilon) = \{q_0\},$$

$L = \{a^n b^n c^n \mid n \geq 1\}$ in ll transition.

Definitions

- $A = \{A_1, A_2, \cdots, A_n\}$, $B = \{B_1, B_2, \cdots, b_m\}$, $A_i, B_j \in 2^V$
- A set $S \subseteq V$ is said to be *attainable* from A and B if

$$S = S_1 * S_2 * \cdots * S_k$$

where

$$S_1 \in A, S_i \in A \cup B, i \geq 2 \text{ and}$$

$$\text{if } S_i \in A \text{ then } * \text{ preceding it is } \cup \text{ or else } * \text{ is } -$$

The set of all attainable sets is denoted by $\mathcal{A}_V(A, B)$. Note that the evaluation is from left to right.

- A run on BBA is defined as the finite sequence of states $q_0 q_1 q_2 \cdots q_k$ where q_0 is the start state, $q_i \in Q, 1 \leq i \leq k$, $q_k \in Q_{accept} \cup Q_{reject}$ and there exists $a \in V$ such that $q_i \in \delta(q_{i-1}, a)$
- A run is called k-run if k is the the length of the run.
- A run is said to be a block run if $q_1 \in Q_{block}$, and $q_k \in Q_{unblock}$ with $\beta_b(q_1) = \beta_{ub}(q_k) = X$.

Contd..

simple unblocking scheme: $\forall q \in Q_{unblock},\ \beta_{ub}(q) \subseteq \beta_b(p)$ for some $p \in Q_{block}$.

useful blocking scheme: at no time the automaton tries to block an already blocked symbol.

perfect unblocking scheme: $\forall q \in Q_{unblock},\ \beta_{ub}(q) = \beta_b(p)$ for some $p \in Q_{block}$.

A BBA is said to be *well-formed* BBA if it follows both useful blocking and perfect unblocking.

Note: perfect unblocking scheme implies simple unblocking.

Notations

- If the affinity relation R is empty then the system is denoted by BBA_{np}
- If the system reads only one symbol at a time then the BBA system is called as a simple BBA system and is denoted by $SBBA$
- The systems by $X_{y,D}$ and the set of languages by $x_{y,D}$ where $X \in \{BBA, SBBA\},\ y \in \{p, np\},\ x \in \{bba, sbba\},\ D \in \{l, ll\}$

Known Results

- BBA_p and BBA_{np} have the same acceptance power.
- The acceptance power of BBA is equivalent to that of $SBBA$.
- For every $L \in BBA_l$ there exists BBA_{ll}, $\mathcal{P}$ such that $L(\mathcal{P}) = L$.
- Given a $BBA, \mathcal{P}$ we can construct an equivalent well-formed $BBA, \mathcal{P}'$ with $L(\mathcal{P}) = L(\mathcal{P}')$.

Complexity Issues

- **Blocking number** denoted by $n(\mathcal{P})$ is defined as the cardinality of the set

  $$\mathcal{A}_V(\beta_b(Q_{block}), \beta_{ub}(Q_{unblock})),$$

  Note that the value of $n(\mathcal{P})$ lies between $1 \leq n(\mathcal{P}) \leq \mid 2^V \mid$.

- **Blocking instant** denoted as $B(\mathcal{P})$ is defined as

  $$B(\mathcal{P}) \ = \ Max\{Card(A) \mid A \in \mathcal{B}(\mathcal{P})\}$$

- **Blocking quotient** of a set $X \subseteq V$ is defined as the length of the longest run from the blocking of X to the unblocking of X. It is denoted by $BQ_X(\mathcal{P})$.

- **Blocking quotient** of $\mathcal{P}$ is defined as $BQ(\mathcal{P}) \ = \ Max\{BQ_X\}$ where the maximum is taken over all the sets X (where $X \subseteq V$ such that there exists $q \in Q_{block}$ with $\beta_b(q) \ = \ X$). We denote blocking quotient simply as BQ if $\mathcal{P}$ is understood.

- $\mathcal{P}(k, m, n)$ denoted a BBA $\mathcal{P}$ with k the blocking number, m the blocking instant and n blocking quotient.

- For every BBA with infinite blocking quotient there is an equivalent BBA with finite blocking quotient

Hierarchy Results

- $REG \subset bba_l(*, 1, *) \subset bba_l(*, 2, *) \subset bba_l(*, 3, *) \subset \cdots\cdots$
- $REG \subset bba_D(1, *, *) \subset bba_D(2, *, *) \subset bba_D(3, *, *) \subset \cdots\cdots$

$$L_k \ = \ \{a_1^n a_2^n \cdots a_k^n \mid n \geq 1\}, \ k \geq 2.$$

- $REG \subset bba_{ll}(*, 1, *) \subset bba_{ll}(*, 2, *) \subset bba_{ll}(*, 3, *) \subset \cdots\cdots$

$$L_k \ = \ \{(a_1 a_2 \cdots a_k)^n b^n \mid n \geq 1\}, \ k \geq 1$$

Contd..

- $REG \subset bba_l(*, 1, *) \subset bba_l(*, 2, *) \subset bba_l(*, 3, *) \subset \cdots\cdots \subset k - NFA$
- $REG \subset bba_l(1, *, *) \subset bba_l(2, *, *) \subset bba_D(l, *, *) \subset \cdots\cdots \subset k - NFA$
- $bba_D(*, *, 1) \ = \ bba_D(*, *, k), \ k \geq 2, \ D \in \{l, ll\}$

Conclusion

- We introduce some complexity measures for BBA - blocking number, blocking instant and blocking quotient
- We also study about the hierarchical structures of BBA arising out of these complexity measures
  - Blocking number and blocking instant gives an infinite hierarchy within BBA in both l and ll.
  - We show that for every BBA with a finite blocking quotient we can construct an equivalent BBA with blocking quotient equal to *one* in both l and ll transitions.